

# 2018 WALTON MIDDLE SCHOOL MATHFEST

Written round: 45 minutes

25 Questions

**Do not open this packet until your proctor instructs you.**

There is no guessing penalty on this test. Each question that is answered correctly will be worth 6 points. The score on this test will be added to the ciphering score to determine the total score.

**No calculator is allowed on this test.** Rulers, protractors, or any other aid are also not permitted on this test.

Scratch paper will be provided. If you need extra scratch paper during the test, raise your hand and ask your proctor.

A time warning will be given when there are 5 minutes left. Be careful to use your time efficiently.

*Good luck and may the odds (and evens) be ever in your favor!*

1. Kerry slept for 6 hours and 115 minutes. How many hours did he sleep?
  - a.  $6 \frac{11}{12}$  hours
  - b.  $7 \frac{11}{12}$  hours
  - c.  $7 \frac{3}{20}$  hours
  - d.  $6 \frac{3}{20}$  hours
  - e. None of the above
  
2. Bret rolls two standard 6-sided dice. What is the probability that the sum of the two numbers he rolls is at least 10?
  - a.  $\frac{1}{12}$
  - b.  $\frac{5}{36}$
  - c.  $\frac{1}{6}$
  - d.  $\frac{5}{6}$
  - e.  $\frac{1}{18}$
  
3. Samantha wants to buy snacks at the zoo. The ticket to get into the zoo costs \$15, and each snack costs \$3.50 each. If Samantha has \$30, and she must purchase ticket to get into the zoo, what is the maximum number of snacks can she buy?
  - a. 7
  - b. 6
  - c. 5
  - d. 4
  - e. 3
  
4. September 29, 2018 is a Saturday. What day of the week will September 30, 2020 be?
  - a. Sunday
  - b. Monday
  - c. Tuesday
  - d. Wednesday
  - e. Thursday
  
5. Let  $(x, y)$  be the solution to the system of equations  $5x + 2y = 10$  and  $-y - x = -2$ . What is  $x + y$ ?
  - a. 1
  - b. 0
  - c. -1
  - d. -2
  - e. None of the above

6. How many distinct factors does 60 have?
- a. 12
  - b. 10
  - c. 8
  - d. 6
  - e. None of the above
7. How many different ways can Maggie order 5 distinct books on a bookshelf?
- a. 5
  - b. 120
  - c. 32
  - d. 60
  - e. None of the above
8. A triangle ABC has integer side lengths  $x$  and  $2x$ . What is the maximum integer value of the last side if the perimeter is equal to 59?
- a. 56
  - b. 29
  - c. 14
  - d. Not enough information
  - e. None of the above
9. The product of three consecutive integers is 2184. What is the sum of the 3 consecutive integers?
- a. 39
  - b. 36
  - c. 33
  - d. 30
  - e. None of the above
10. Amanda can finish a test in 30 minutes. Alice can finish the same test in 45 minutes. If both Amanda and Alice work together, how long will it take them to finish the test?
- a. 20 minutes
  - b. 19 minutes
  - c. 18 minutes
  - d. 17 minutes
  - e. None of the above

11. Let  $N$  be the smallest number with the following properties:

- $N$  is a perfect square
- $N$  has 9 distinct factors
- $N$  is not divisible by 6

What is  $N$ ?

- a. 100
- b. 196
- c. 225
- d. 144
- e. None of the above

12.  $38!$  (or 38 factorial) is shorthand notation used to represent  $38 \times 37 \times 36 \times \dots \times 3 \times 2 \times 1$  (where  $*$  means multiplication). Exactly, how many zeros are at the end of  $38!$  ?

- a. 8
- b. 7
- c. 6
- d. 5
- e. 4

13. How many distinct diagonals does a regular dodecagon have?

- a. 54
- b. 55
- c. 64
- d. 108
- e. None of the above

14. The greatest common factor of two numbers is 30. The least common multiple of those same two numbers is 630. What is the product of the two numbers?

- a. 21
- b. 18900
- c. 18000
- d. 600
- e. None of the above

15. What is the unit's digit of  $2018^{2020}$  ?

- a. 8
- b. 4
- c. 2

- d. 6  
e. None of the above
16. A rectangle has a length that is 2 units less than 3 times greater than its width. A square has an area that is 84 units more than the area of the rectangle. If the diagonal of the square is  $10\sqrt{2}$ , what is the width of the rectangle?
- a. Not possible  
b.  $8/3$   
c. 3  
d.  $10/3$   
e. None of the above
17. Three fair 6-sided dice are rolled. What is the expected value of the sum of the three numbers that face up?
- a. 7  
b. 9  
c. 14  
d. 18  
e. None of the above
18. What is the area of the quadrilateral with vertices  $(2,0)$ ,  $(1,8)$ ,  $(-4,0)$ , and  $(1, -64)$ ?
- a. 72  
b. 144  
c. 216  
d. 432  
e. None of the above
19. Simplify the expression  $\frac{(\square^2-1)(\square^3+2\square^2+2\square+1)}{(\square^3-1)(\square+1)^2}$ .
- a.  $x^2 + 1$   
b.  $x^2 - 1$   
c.  $x - 1$   
d.  $x + 1$   
e. 1
20. Prowlong the fast cat takes the same amount of time to walk 5 miles and run 10 miles. If Prowlong runs at 5 miles per hour faster than he walks, then how many minutes does it take him to walk 15 miles?
- a. 180

- b. 120
- c. 60
- d. 30
- e. None of the above

21. If  $\square = \sqrt{-1}$ , then which of the following does  $\square^{2018}$  equal?

- a.  $\square$
- b.  $-\square$
- c. 1
- d. -1
- e. None of the above

22. If  $r$  and  $s$  are the roots of the  $\square(\square) = \square^2 - 10\square + 18$ , then what is  $|\square - \square|$ ?

- a.  $\frac{\sqrt{7}}{2}$
- b.  $\frac{11}{2}$
- c.  $3\sqrt{3}$
- d. 5
- e.  $2\sqrt{7}$

23. How many three-digit numbers are divisible by their hundreds digit?

- a. 240
- b. 280
- c. 320
- d. 360
- e. None of the above

24. Equilateral triangle  $\square\square\square$  is rotated  $60^\circ$  about its center to form triangle  $\square'\square'\square'$ . What is the area of the intersection of the two triangles if the circumradius of triangle  $\square\square\square$  is 1?

- a.  $\frac{1}{2}$
- b. 1
- c.  $\frac{\sqrt{3}}{2}$
- d.  $\sqrt{3}$
- e. None of the above

25. How many ways are there to split the letters in "waltonmathfest" into 3 "words" where each "word" must have at least length 1 and must contain a vowel (order of the words matters too)?

- a.  $\left(\frac{16!}{8!}\right)$
- b.  $\left(\frac{16!}{7!}\right)$
- c.  $\left(\frac{16!}{6!}\right)$
- d.  $\left(\frac{16!}{5!}\right)$
- e.  $\left(\frac{16!}{4!}\right)$

Answer Key:

- 1. B
- 2. C
- 3. D
- 4. D
- 5. E
- 6. A
- 7. B
- 8. B
- 9. A
- 10. C
- 11. A
- 12. A
- 13. A
- 14. B
- 15. D
- 16. B
- 17. E
- 18. C
- 19. E
- 20. A
- 21. D
- 22. E
- 23. B
- 24. C
- 25. D